Mahila Vikas Sanstha's

**INDRAPRASTHA NEW ARTS COMMERCE & SCIENCE COLLEGE,** AT POST NALWADI, DIST. WARDHA (M.S.)

Accredited 'B' by NAAC

Approved by government of Maharashtra

Affiliated to Rashtrasant Tukadoji Maharaj Nagpur University, Nagpur

Recognised by U.G.C New Delhi under section 2 (f) & 12 (b) of UGC act 1956

1) Theorem : Cauchy theorem for abelian group.
2) State and prove first sylow theorem.
3) Statement: let G be the finite group and let p be a prime then all sylow p subgroup of G are conjugate and their number np divides o(G) and stastifies $n_p$=l(mod p).
4) Prove that a sylow p-subgroup of a finite group G is unique iff it is normal.
5) If o(G)=pq, p and q are distinct primes and p<q. Show that if p does not divide (q-1),then G is Cyclic.
6) Show that every group of order $p^2$, p is prime is either cyclic or is isomorphic to the direct product of two cyclic groups each of order p.
7) Example: There is no simple group of order 63,56,36.
8) Example: Let o(G)=30,Show that,
   I. Either sylow 3-subgroup or sylow 5-subgroup is normal in G.
   II. G has a normal subgroup of order 15.
   III. Both sylow 3-subgroup and sylow 5-subgroup are normal in G.
9) Show that the set an of all even permutation of Sn is a normal subgroup of Sn and o(An)=n!/2.
10) The alternative group An is simple if n>4, consequently, sn is not solvable if n>4.
11) Let $H_1$ & $H_2$ be normal in G then G is an internal direct product of $H_1$ & $H_2$ iff
    (a) G = $H_1$ $H_2$
    (b) $H_1$ n $H_2$ ={e }
12) If a group of order $p^n$ contains exactly one subgroup each of order P,$P^2$,….. $P^{n-1}$ then it is cyclic.
13) Let A and B be finite cyclic group of order m & n resp. Prove that A*B is cyclic iff m & n are relatively Prime (i.e.(m,n)=1).
14) If G be group & suppose G is internal direct product of $H_{1,}H_2$, …., $H_N$. Let T be External direct product of $H_{1,}H_2$, …., $H_N$ then G & T are isomorphic.

15) Theorem : Let $H_1, H_2, \ldots H_n$ be normal in G , then G is on IDP of $H_1, H_2, \ldots H_n$ iff (a) $G=H_1, H_2, \ldots H_n$

(b) $H_i \cap H_j = \{e\}$ $i \neq j$, $Gi=1,2,\ldots,n$ or $H_i \cap H_j, H_2, \ldots H_{i-1}, H_{j+1}, \ldots H_n) = \{e\}$.

16) Let A Be finite abelian group . Then there exists a unique list of integers $m_1, m_2, \ldots, m_k$(all>1) such that

$|A| = m_1, \ldots, m_k$, $m_1 | m_2 | \ldots | m_k$,

And $A = C_1 \oplus \ldots \oplus C_K$ , where C1,.....,CK are cyclic subgroups of A of order $m_1, \ldots, m_K$ resp, consequently, $A = Zm_1 \oplus \ldots \oplus Zm_K$.

17) Theorem : If E is a finite Extension of a field F, then |G(E/F)|<=[E:F].

18) Define Automorphism and fixed field

19) Example: G=G(C/R) Let G1=G(C/R) then prove that |G1|=Z.

20) Prove Lemma (Dedekind)

21) Define Automorphism group

22) Theorem: Let $H = \{e=g, \ldots g_n\}$ & Let $[E:E_H]=m$. If possible ,suppose m<n. Let $\{a, \ldots a_m\}$ be a basic for E over $E_H$.

23) Theorem: Let E be a finite separable extension of a field F Then the following are equivalent.

I. E is a normal extension of F.
II. F is the fixed field of G(E/F)
III. [E:F] = |G(E/F)|.

24) State Fundamental theorem of Galois theory.

25) Theorem: Every polynomial $f(x) \in c[x]$ factors into linear factor in c[x].

26) Definition of Galois group

27) Define Integral domain.

28) Let R be an integral domain with unity $a, b \in R$ be two non_zero elements s.t. a|b or b|a ,a and b are asdociates and conversely.

29) Define Prime element

30) Theorem: An irreducible element in a commutative principle ideal domain(PID) is always prime element.

31) If R is UFD, then the factorization of element in R as a finite product of irreducible factors is unique to within order and unit factor.

32) Define Associates element

33) Let R be UFD and a,b $\in$ R then there exist an greatest common divisor of a & b that is unique determined to within an orbitory unit factors.

34) Theorem: Every PID is a UFD but UFD is not necessary a PID.

35) Define Irreducible element

36) Theorem: Every Euclidean domain is a PID.

37) Theorem: Every Euclidean domain is a UFD.

38) Prove that CID R={a+b$\sqrt{-s}$/a,b $\in$ z}is not a UFD.

39) Define Unique factorization domain.

40) Theorem: Let R be a unique factorization domain. Then the poly ring R[x] over R is also a unique factorization domain

41) Theorem: Let R =F[x] be a poly ring over a commutative integral domain F. Let f(x) and g(x) $\neq$ 0 be poly in F[x] of degrees m & n repectively. Let K= max(m-n+1,0) and 'a' be the leading coeff of g(x). then unique poly q(x)&r(x)

in F[X] s.t $a^k(f)=q(x).g(x)+r(x)$ wherenr(x) has degree less than the degree of g(x).