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1. By using ∈-δ technique of limit of a function, show that $\lim_{x\to 1} 3x2 + x = 4$.
2. Examine the continuity of the function f defined as :

$$f(x) = \begin{cases} 2 + x & , x \leq 1 \\ 4 - x & , 1 < x \leq 2 \\ -2 + 3x - x2 & , x > 2 \end{cases}$$

at the points x = 1 and x = 2.

3. Let f(x) = x · sin(1/x) ,x ≠ 0

= 0 , x = 0

Show that f is continuous but not differentiable at x = 0.

4. If y = a cos(log x) + b sin(log x), then show that :

$$x^2 y_{n+2} + (2n + 1)x\, y_{n+1} + (n^2 + 1)y_n = 0.$$

5. Prove by Maclaurin's theorem, that:

$$e^x.\log_e(1+x) = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

6. Find the radius of curvature for the cycloid x = a(t + sin t), y = a(1 – cos t) at t = π/2.
7. Find the asymptotes of the curve:

$$y^3 - 5xy^2 + 8x^2y - 4x^3 - 3y^2 + 9xy - 6x^2 + 2y - 2x = 1.$$

8. Evaluate :

1. $\lim_{x\to 0} x \tan\left(\frac{\pi}{2} - x\right)$

2. $\lim_{x\to 0} (cosec\, x)^{\frac{1}{logx}}$

9. If u = $\log_e \sqrt{(x^2 + y^2 + z^2)}$ then prove that :

$$(x^2 + y^2 + z^2)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1.$$

10. If z = f(x, y) and x = $e^u - e^{-v}$, y = $e^{-u} + e^v$, then prove that :

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial u} = x\frac{\partial z}{\partial u} - y\frac{\partial z}{\partial u}$$

11. If z is a homogeneous function of x and y of degree n, then prove that:

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz.$$

12. If x = r sin θ cos φ, y = r sin θ sin φ and z = r cos θ, then show that :

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = r^2 sin\theta$$

13. Evaluate :

$$\int \frac{2x+5}{\sqrt{x^2+3x+1}}$$

14. Show that :

$$\int_0^1 \frac{(1-4x+2x^2)}{\sqrt{2x-x^2}}dx = 0$$

15. Prove that :

$$\int sec^n\, xdx = \frac{sec^{n-2}x.\tan x}{n-2} + \frac{(n-2)}{(n-1)}\int sec^{n-2}x\, dx$$

and hence evaluate $\int \sec^6 x\, dx$.

16. Evaluate :

$$\int_0^\pi \frac{x\, dx}{a^2cos^2x + b^2sin^2x}$$

17. Show that $f(x) = \frac{1}{1-e^{\frac{1}{x}}}$, $x \neq 0$ has a simple discontinuity at x = 0

18. If y = sin (ax + b), then prove that:

$y_n = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$.

19. Expand log x in powers of (x – 1) upto the terms in x^2.

20. Find the radius of curvature of the curve s = c log sec ψ.

21. If z = sin xy and x = 2t + 5, y = 3t^2 , find $\frac{dz}{dt}$.

22. If u = 2x + 3y; v = 5x + 6y, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

23. Evaluate :

$$\int_0^{\frac{\pi}{2}} \sin^5 x \, . \cos^4 x \, dx$$

24. Find :

$$\int \frac{dx}{\sqrt{x^2-2x+5}}$$

25.If $\cos^{-1}\frac{y}{b} = \log\left(\frac{x}{n}\right)^n$, prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0.$$

26.Prove that

$$\log secx = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \cdots$$

27.State and prove the all form of Taylor's Theorem.

28.If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, $x \neq y$ then show that

i. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

ii. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u.$

29.If $v = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = m(m+1)r^{m-2}$$

30. Find the envelope of the lines $\frac{x}{a} + \frac{y}{b} = 1$, when the parameters a and b are connected by the relation $a + b = c$.

31. If the $x = r\sin\theta\cos\emptyset$, $y = r\sin\theta\sin\emptyset$. $z = r\cos\theta$,

Show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)} = r^2\sin\theta$

32. Verify $JJ' = 1$, if $x = u(1-v)$, $y = uv$.

33.Expand $\sin xy$ in power s of $(x-1)$ and $\left(y-\frac{\pi}{2}\right)$ upto second degree terms.

34.Divide 24 into three parts such that the continued product of the first, square of second and cube of the third is maximum.

35.Find the maximum and minimum distance of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$.

36.If $u_n=\int_0^{\frac{\pi}{2}}\theta\sin^n\theta\,d\theta$ and $n>1$, prove that

$$u_n=\frac{1}{n^2}+\frac{n-1}{n}u_{n-2}.$$ Hence deduce that $u_5=\frac{149}{225}$.

37. Obtain a reduction formula for $\int x^m(\log x)^n\,dx$ and use it to evaluate $\int_0^1 x^4(\log x)^3\,dx$.

38. Evaluate $\int\frac{dx}{(x-1)^2(x-2)(x^2+4)}$.

39.If $I_n=\int_0^a(a^2-x^2)^n\,dx$ and $n>0$, prove that $I_n=\frac{2na^2}{2n+1}I_{n-1}$