Mahila Vikas Sanstha's
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Mathematics Paper II , Sem V (Metric space ,Boolean Algebra & Graph Theory)

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1) Prove that the set of even natural numbers is denumerable(or countable).
2) Every finite subset of a countable set *A*. Then *A* is also finite.
3) Prove that the product set $J \times J$ is countable, where *J* is the set of natural numbers.
4) Let *X* be a non-empty set and $d : X \times X \rightarrow R$ satisfies the condition:
   i) d(x,y) = 0 iff x=y ii)d(x, y)≤ d(x, z) + d(y, z), then show that d is a metric on X.
5) In any metric space X, the empty set ϕ and the full space X are open sets.
6) In any metric space X, each open sphere is an open set.
7) Every non-empty open set on the real line is the union of a countable disjoint class of open intervals.
8) In any metric space X, the empty set ϕ and the full space X are closed set.
9) Let X be a metric space. A subset F of X is closed if and only if its complement F' is open.
10) In any metric space X, each closed sphere is a closed set.
11) Every convergent sequence in a metric space is a Cauchy Sequence.
12) $\{x_n\}$ is a Cauchy Sequence of a real numbers iff $\{x_n\}$ is convergent in R.
13) Let Y be the subspace of a complete metric space X. Then Y is a complete iff Y ic closed.
14) Show that E = (0, 1) is not compact.
15) Compact subsets of metric spaces are closed.
16) Closed subsets of compact sets are compact.
17) If E is an infinite subsets of a compact set K, then E has a limit point in K.
18) Every K-cell is compact.
19) A set E in R is connected iff E is one of the interval.

20) (Weirstrass Theorem) Every bounded infinite subset of R^k has a limit point in R^k.

21) Show that the relation ' Divides' defined on the set of Natural Numbers n is a partial order relation .

22) Define join operation and Meet operation.

23) Does the lattice (S_{12}, D) is complimented Lattice?

24) Every Chain is a Distributive Lattice.

25) The direct product of any two distributive lattice is a distributive lattice.

26) In a distributive lattice , the compliment of an element is unique.

27) What is homomorphism?

28) Show that the lattice (S_n ,D) for n = 36 is isomorphic to direct product of lattice (S_n ,D) for n = 4 and 9.

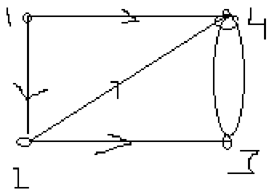
29) State the Boolean identities and prove them.

30) Let A = { 3,4,5,6,7,8} and R = {(x – y) is divisible by 3} show that R is equivalence relation.

31) Show that a complete digraph with n nodes has the maximum numbers of edges it has is in n(n - 1), assuming loop.

32) In a simple digraph , the length of any elementary path is less than or equal to (n - 1) ,where n is the nymber of nodes in the graph . Similarly, the length of elementary does not exceed n.

33) Find the node-base for following diagraph.



34) In a simple diagraph G = < V, E> . Every node of the digraph lies in exactly one strong components.

35) Show the path matrix can be used to obtained strong component containing any particular node of the graph.

36) Show that the path matrix of a digraph can be used in determining whether certain procedure in a program are recursive.

37) For the digraph determine A', AA', and A'A . Unterpret the entries of the matrix A ∧ A'.(A' is a transpose of A)

38) For any n × n Boolean matrix A, show that $(I + A)^{(2)} = (I + A) \wedge (I + A) = I + A + A^{(2)}$. Where I is the n × n identity matrix and $A^{(2)} = A \wedge A'$. show that for positive integer r $(I + A)^{(r)} = (I + A) \wedge (I + A) \ . \ . \ . \wedge (I + A) = I + A + A^{(2)} +. . .+ A^{(r)}$

39) In reference to Graph theory , define a path and a cycle.

40) Define a directed tree.