QUESTION BANK
Msc MATHEMATICS 1ST YR SEM II

1) Prove that there is no rational number whose square is 12.
2) Under what conditions does equality hold in the Schwarz inequality?
3) If z is a complex number , prove that there exists an r>0 and a complex number w with |w|=1 such that z=rw . Are w and r always uniquely determined by z ?
4) If x , y are complex , prove that ||x|-|y||<|x-y|.
5) Prove that no order can be defined in the complex field that turns it into an ordered field.
6) Prove Proposition.
7) Prove that the empty set is a subset of every set.
8) Prove that there exist real numbers which are not algebraic.
9) Is the set of all irrational real numbers countable ?
10) Are closures and interiors of connected sets always connected ?
11) Prove that every compact metric space K has a countable base, and that K is therefore separable .
12) Prove that every open set in R1 is the union of an at most countable collection of disjoint segment.
13) Prove that the Cauchy product of two absolutely convergent series converges absolutely.
14) Definition 3.21 can be extended to the case in which the a_n lie in some fixed R^k.Absolute convergence is defined as convergence of$\sum |a_n|$.Show that Theorems 3.22,3.23,3.25(a),3.33,3.34,3.45,3.47, and 3.55 are true in this more general setting. (only slight modification are required in any of the proofs.)

15) Prove that If $\sum a_n$ =A, and $\sum b_n$ =B, then $\sum(a_n + b_n)$=A+B, and $\sum ca_n$ =Ca, for any fixed c.
16) Let f be a real uniformly continuous function on the bounded set E in R^1 .prove that f is bounded on E. Show that the conclusion is false if boundedness of Eis omitted from the hypothesis.
17) A uniformly continuous function of a uniformly continuous function is uniformly continuous. State this mc :cisely and prove it.
18) Suppose f is a real function defined or. ... satisfies.
19) If f (x) = $|x|^3$,compute f'(x),f"(x) for all real x, and show that $f^{(3)}$ (0) does not exist
20) Let f be defined for all real x, and suppose that $|f(x)-f(y)\leq (x-y)^2$.
21) Formulate and prove an inequality which follows from Taylor's theorem which remains valid for vector-valued function.
22) Prove that every uniformly convergent sequence of bounded function is uniformly bounded.
23) State and prove Parseval's theorem.
24) Prove that Let r be a positive integer . If a vector space X is spanned by a set of r vectors , then dim X≤r.
25) If S is a non empty subset of a vector space X , prove (as asserted in sec.9.1) that the span of S is a vector space.
26) Assume A £ L(X, Y) and Ax =0 only when X = 0. Prove that A is then 1-1.
27) Suppose that f is a real valued function defined in an open set E¢R^n , and that f has a local maximum at a point X £ E. Prove that f' (x) =0.
28) Show that the existence (and even the continuity) of D_{12} f does not imply the existence of D_1 f . For example let f(x,y) =g(x), where g is nowhere differentiable .
29) Give a similar discussion for f(x,y) = $2x^3 + 6xy^2 - 3x^2 + 3y^2$.
30) State and prove the Lebesgue's dominated convergence theorem.
31) Prove that the function F given by (96) is continuous on [a , b].

32) Prove that a complex function f is measurable if and only if $f^{-1}(V)$ is measurable for every open set V in the plane.

33) (a) Show that the simplex Q^k is the smallest convex subset of R^k that contains $0, e_1, \ldots, e_k$.(b) show that affine mappings take convex sets to convex sets.

34) Answer analogous questions for the mapping defined by $u = x^2 - y^2$, $v = 2xy$.

35) Take n = m = 1 in the implicit function theorem , and interpret the theorem (as well as its proof) graphically.

36) If f (x) = 0 for all irrational x, f(x) = 1 for all rational x , prove that f€R on [a , b] for any a < b.

37) Let f be a continuous real function on R^1, of which it is known that $f'(x)$ exists for all $x \neq 0$ and that $f'(x) \to 3$ as $x \to 0$. Does it follow that $f'(0)$ exists?

38) Let X be the metric space whose points are the rational numbers , with the metric $d(x, y) = |x - y|$. what is the completion of this space?

39) Prove that the Cauchy product of two absolutely convergent series converges absolutely.

40) If $\{f_n\}$ is a sequence of measurable functions , prove that the set of points x at which $\{f_n(x)\}$ converges is measurable.