

Question Bank (2023-24)
Subject –Mathematics I (Sem IV)
(Real Analysis)

Unit-I

- 1) Define-Closed Interval ,Open Interval, Semi Closed and semi open interval.
- 2) Define- Bounded Set ,Unbounded Set,Bounded Above,Bounded Below.
- 3) Theorem- Let X be a non-empty subset of \mathbb{R} which is bounded above. Then a real number M is the supremum of X if and only if
 - (a) $x \leq M$, for all $x \in X$
 - (b) For each real number $\epsilon > 0$, there is a real number $x \in X$ such that $x > M - \epsilon$.
- 4) Let $X < Y \subseteq \mathbb{R}$ be non empty bounded sets and $\alpha \in \mathbb{R}$. Then
 - a) $\sup(X+Y) = \sup X + \sup Y$
 - b) $\inf(X+Y) = \inf X + \inf Y$
- 5) Define Neighbourhoods. Prove that an open interval is a nbd of each of its pair.
- 6) Prove that a finite set is not a nbd of any of its points.
- 7) Define Open set. The union of an arbitrary collection of open set.
- 8) Theorem- Every Infinite bounded subset of \mathbb{R} has a limit point.
- 9) Derived set of a set X is closed.

Unit-II

- 1) Theorem on Convergent Sequence.
- 2) Every convergent sequence has a unique limit.
- 3) Show that the sequence $\langle x_n \rangle$, Where $x_n = \frac{1}{2n}$ converges to 0.
- 4) Define-Operation on sequence, constant sequence.
- 5) Algebraic properties of convergent sequence.
- 6) Theorem-sandwich Theorem.
- 7) Theorem-A Monotonic increasing sequence is convergent, if and only if it is bounded.
- 8) Theorem on Cauchy sequence.

Unit-III

- 1) Theorem –Necessary and sufficient condition of a series
- 2) The Series $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ is not convergent.
- 3) The series $1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2n-1} + \dots$ is not convergent.
- 4) Theorem-If $\sum x_n$ and $\sum y_n$ be two given positive term series such that $\frac{x_n}{y_n} = l \neq 0$, where l is finite, then the two series $\sum x_n$ & $\sum y_n$ are either both convergent or divergent.
- 5) Theorem- Cauchy's Root Test.

6) Theorem- D'Alembert's Ratio Test.

Unit-IV

- 1) Properties of Integral equation.
- 2) If M and m be the supremum and infimum of a bounded function f on $[a,b]$ then $m(b-a) \leq L(p,f) \leq U(p,f) \leq M(b-a)$.
- 3) Theorem (Darboux's theorem)
- 4) A bounded monotonic function f defined on $[a,b]$ is integrable on $[a,b]$.
- 5) Mean Value Theorem for integral.