Mahila Vikas Sanstha's
INDRAPRASTHA NEW ARTS COMMERCE & SCIENCE COLLEGE, AT POST NALWADI, DIST. WARDHA (M.S.)
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BACHERLOR OF SCIENCE SEMESTER 3

PAPER 2- MODERN ALGEBRA

1. Show that the set of all even integers (including zero) w.r.t addition is an infinite abelian group.
2. show that the set of all numbers cos ϴ+ sin ϴ from an infinite abelian group w.r.t ordinary multiplication ; where ϴruns over all rational numbers.
3. Let G consist of the real number 1,-1 .show that is an abelian group of order 2 under the multiplication of real numbers.
4. Show that the set of cube roots of unity forms an abelian group w.r.t the usual multiplication of numbers.
5. Prove that the set G ={0,1,2,3,4} is a finite abelian group of orders 5 w.r.t addition modulo 5.
6. Let G be a group and a^{-1} b^{-1} ab=e¥ a,b£ G. prove that G is an abelian group.
7. Prove by giving an example that the union of subgroup is not necessarily a subgroup.
8. If S ={0,2,4} then show that (s+6) is a subgroup of the group(I6+6).
9. Every cyclic group is abelian.
10. The order of cyclic groups is same as the orders of its generator.
11. Show that there exist only two generators of any infinite cyclic group.

12. If H is finite subgroups of G ,then the numbers of element in a right (or left) coset of H is equal to the order of H.
13. If P is prime number and a is any integer ,then a^p=a mod p.
14. If H and K are two subgroup of a group G ,then prove that HK is subgroup of G if and only if HK=KH.
15. The subgroup N of a group G is normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.
16. Prove that every subgroup of an abelian group is normal.
17. If H is a subgroup of G and N is normal subgroup of G , show that $H \cap N$ is normal subgroup of H.
18. If an abelian group G is simple ,then show that the order of G is prime .
19. Show that every quotient group of an abelian group is abelian but its converse is not true .
20. Let G ={ $a, a^2, a^3, a^4, a^5, \ldots, a^{12}=e$} be a cyclic group under multiplication .Let G ={$a^2, a^4, a^6, \ldots, a^{12}$} be its subgroups then prove that the mapping $a^n \rightarrow a^{2n}$ is a homomorphism of onto G.
21. Prove that every isomorphic image of cyclic group is cyclic.
22. The relation of isomorphism in the set of group is an equivalence relation.
23. The product of disjoint cycles is commutative.
24. Every permutation can be expressed as product (composite) of disjoint cycle .
25. Every permutation can be expressed as a product of transpositions .(2- cycles).
26. Write down all permutation on three symbols a,b,c. which of these are even?

27. Determine for what m an cycle is an even permutation?
28. Write the permutation in example 2 as the product of disjoint cycles.
29. Show that the set R of integer mod 7 under addition and multiplication mod 7 is a commutative ring with unity.
30. Let R be the set of integers mod 7 under addition and multiplication mod 7 is a commutative ring with unity .
31. Let R be the set of integers mod 6 under addition and multiplication mod 6 ,then R is commutative ring with unit element.
32. Show that every field is an integral domain.
33. If F is a field , then prove that its only ideals are (0) and F itself.
34. If U is an ideal of R ,then R/U is a ring .
35. Let R be a commutative ring with unit element whose only ideals are {0} and R itself .then R is a field.
36. Show by an example that extension ring of an integral domain need not essentially be an integral domain.
37. Show that ring of integral is Euclidean ring.
38. Every ideal in a Euclidean ring is principal ideal.
39. Prove that the necessary and sufficient condition that element a in Euclidean ring is unit is that d(a)=d(1).
40. If F is a field ,every ideal in F[x] is principal ideal ring .