Mahila Vikas Sanstha's

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1. A particle moves along the curve $x = t^3 + l$, $y = t^2$, $z = 2t^2 + 5$ at any time t. Find the components of velocity and acceleration at t — 1 in the direction $2\hat{I} + j + 3k$

2. Find the directional derivative of $\emptyset i = x^2y^2 + yz^2$ at the point (2, -l, 1) in the direction of the vector +2 j — 2k

3. Prove that div grad $r^{a} = n(n + l)r^{n-2}$.

4. Find the total work done in moving a particle in a force field given by = 2xyi + 3zj + 6xk along the curve $x = t^2 + t$, $y = t$, $z = t^3$ from t = O to t = 1

R

5. Evaluate $\int_0^c \int_x^c \frac{xdydx}{\sqrt{x^2 + y^2}}$, c is a positive constant, by changing the order of Integration.

6. Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} y^2 \sqrt{x^2 + y^2}\, dxdy$ 3 2 2 by changing Into polar coordinates.

7. Evaluate $\iint (x2 + y2)dxdy$ by changing into polar coordinates, where R is the region$(x2 + y2) \leq 1$

8. Evaluate the surface Integral $\iint (yz\hat{I} + zxj + xyk)$ o dS where S is the surface of the sphere$x^2 + y^2 + z^2 = I$ in the first octant.

9. Evaluate $\iiint (x2 + y2 + z2)\ dxdydz$, where R denotes the region bounded by x = O, y = O, z = 0 and x + y + z = a.

10. If $\bar{A} = (2y + 3)i + xzj + (yz — x)k$, then evaluate Ä o di along the straight lines from(O, O, O) to (O, O, 1), then to (O, 1, l) and then to (2, l, 1).

11. Find the work done in moving particle in a force field $F = 3x^2 i + (2xz — y) j + zk$ along the straight line from (O, O, O) to (2, 1, 3). 6

12. Evaluate $\iint$ F o dS, where $F = (x + y^2)i — 2xj + yzk$ and S is the surface of the plane$x + y + z = 3$, which is located in the first octant.

13. Find the volume of the region common to the intersecting cylinder $x^2 + y^2 = a^2$ and$x^2 + z^2 = a^2$

14. Consider a simple closed curve C in a simply-connected region with appropriate properties, Prove that M dx + N dy = O if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ everywhere in the region.

15. Verity Green's theorem in the plane for $\oint(3x2 — 8y2)\, dx + (4y — 6xy)\, dy$, where C is theboundary of the region defined by $y = \sqrt{x}$ and $y = x^2$.

16. Verify the divergence theorem for $A = 4xi — 2y^2 j + z^2 k$ taken over the region bounded by $x^2 + y^2 + z^2 = 4$, z = 0 and z = 3.

17. Evaluate $\iint \overline{\nabla}\, x\, \bar{A}\, o\, \hat{n}\, dS$ where $A = ((x^2 + y - 4)i - 2y^2 j + z^2 k)$ and S is the surface of the hemisphere $x^2 + y^2 + z^2 = 16$ above xy-plane,

18. Verify Stoke's theorem for the vector field defined by $= (x^2 — y^2)\, i + 2xyj$ in the rectangular region in the xy plane bounded by lines x = O, x = a, y = 0, y = b.

19. Evaluate FoñdS by using divergence theorem, where $F = 4xzi — y^2 j + yzk$ and S is the surface of the cube enclosed by x = O, y = O, z = O, x = l, y = 1, z = 1

20. Evaluate $\oint[(xy — y^2)dx + (x^2 — y)dy]$, by using Green's theorem in a plane, where C is the closed curve of the region bounded by y = x and $y = x^2$.

21. Test the convergence of .

(i) $\int_0^1 \frac{\cos x}{x^2}\, dx$

∞

22. Prove that

(i) $\boxed{n+1} = n!$, n = 1, 2, 3, ...

∞ 7 (4)

23. Show that Beta function B(m, n) = $\frac{\lceil m \lceil n}{\lceil m+n}$; m, n > 0 .

24. Show that $\bar{f}$ = (x^2 — yz)i + (y^2 — zx)] + (z^2 — xy)k is irrotational.

25. Evaluate $\int_1^2 \int_0^{y/2} y dy dx$.

26. If H =curl A, then show that $\iint$ H o ñdS = 0 for any closed surface S by using divergence theorem.

27. $\oint_C xdy - ydx$.

28. Evaluate $\int_0^\infty \frac{x^3}{(1+x)^9} dx$

29. Evaluate

30. ..

31. Evaluate $\int_0^1 x^4 (l — x)^3 \, dx$.

32. A particle moves so that its position vector is given by r = cosωti + sinωtj where ω is a constant. Show that

(a)The velocity $\vec{v}$ of the particle is perpendicular to F.

(b)The acceleration is directed towards the origin and has magnitude proportional to the distance from the origin and

(c) $\bar{r} \; x \; \bar{v} = \bar{a}$, a is constant vector.

33. If $\vec{v} = \bar{\omega}$ x $\bar{r}$, prove that $\bar{\omega}$=$\frac{1}{2}$curl $\vec{v}$, where $\bar{\omega}$ is a constant vector.

34. Find the directional derivative of $x^2y^2z^2$ at the point (1, 1, -1) in the direction of the tangent to the curve x = et, y = Sin 2t + l, z = 1- cost at t = 0

35. Evaluate $\iint_{\sqrt{x}}^{1} e^{\frac{x}{y}}$ dydx by changing the order of integration.

36. If $\varphi = 2xz^4 — x^2y$, find $\overline{\nabla}\varphi$ at (2, —2, —l).

37. Verify that the vector $\bar{A} = 3y^4z^2\,i + 4x^3z^2\,j — 3x^2y^2\,k$ is solenoidal.

38. Evaluate $\iint_0^2 (x + 2)\, dxdy$

39. Evaluate $\int_1^2 \bar{F} dt,$ where $\bar{F}$ =3i + (t^3 +4t^7)j + tk.

40. Determine limits for $\iiint x2\ y\ dV$, where V is the closed region bounded by the planes x + y + z = 4, x = 0, y = 0, z = 0.

41. Write the statement of Gauss Divergence Theorem.

42. Suppose $\bar{H}$= curl $\bar{A}$. Prove that $\iint H^{\circ}\bar{n}\, dS = 0$ for any closed surface S.